

MTH 111

Midterm I

Spring 2012

Question 1. Evaluate the following limits

a)

$$\lim_{x \rightarrow -\frac{4}{3}} \frac{9x^2 - 16}{3x + 4}$$

b)

$$\lim_{x \rightarrow 5} \frac{|2x - 10|}{x - 5}$$

c)

$$\lim_{x \rightarrow -\infty} \frac{3x^3 - 2x^2 + 10}{5x^3 - 4x + 16}$$

Question 2. Find the derivatives of the following functions

a)

$$f(x) = (2x^2 - x + 1)^{-11}$$

b)

$$f(x) = \frac{2x^2 - 1}{\sqrt{x} + 1}$$

Question 3. Find $f'(1)$ using the definition of the derivative where

$$f(x) = 3x^2 - x + 1$$

Question 4. Consider the implicit function

$$y^2 + 3xy = 4$$

a) Find $\frac{dy}{dx}$

b) Find the equation of the tangent line to this function at the point $(1, -4)$.

Question 5. Classify the second-degree equation

$$3x^2 - 2y^2 + 6x - 8y - 17 = 0$$

Question 6. Sketch the graph of

$$\frac{(y - 4)^2}{9} - \frac{(x + 2)^2}{16} = 1$$

Question 7.

a) Write the point $(1, -1)$ in polar coordinates.

b) Sketch the graph of the polar curve given by the equation $r = 1 + \sin(\theta)$.

Question 8. The width of a rectangle is decreasing at a rate of 3 cm/min while the height of the rectangle is increasing at a rate of 4 cm/min. At what rate is the area of the rectangle changing when its width is 4 cm and its height is 10 cm ?

Bonus. Find the equations of the tangent lines to the ellipse

$$x^2 + 4y^2 = 36$$

that pass through the point $(12, 3)$.